RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2018-21]
B.A./B.Sc. SECOND SEMESTER (January – June) 2019
Mid-Semester Examination, March 2019

Date : 28/03/2019

MATH FOR ECONOMICS (General)

Time : 11 am – 12 noon

Paper : II

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

(*Answer any two questions*) [2×5]

1. a) Find the point of discontinuity of the function\
$f(x)=[x]+[-x], x\in\mathbb{R}$.

b) A function $f:\mathbb{R}\to\mathbb{R}$ is continuous on $\mathbb{R}$ and $f(x)=0 \forall x\in\mathbb{Q}$. Prove that $f(x)=0 \quad \forall x\in\mathbb{R}$. [3+2]

2. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=?$ [5]

3. A function $f:\mathbb{R}\to\mathbb{R}$ is continuous on $\mathbb{R}$ and $f(x+y)=f(x)+f(y)\forall x,y\in\mathbb{R}$. If f(1) = k, prove that $f(x)=kx\forall x\in\mathbb{R}$. [5]

4. Let $f(x)=\begin{cases} x, & x\in\mathbb{Q} \\ 2-x, & x\in(\mathbb{R}-\mathbb{Q}) \end{cases}$

Show that i) $\lim\limits_{x\to 1} f(x)=1$: ii) $\lim\limits_{x\to c} f(x)$ doesn't exist, if $c\neq 1$. [2.5+2.5]

Group – B

(*Answer any three questions*) [3×5]

5. The intersection of two subspaces of a vector space V over a field *F* is a subspace of V. Prove or disprove this statement. [5]

6. Prove that $\alpha_1=(1,0,0)$, $\alpha_2=(1,1,0)$, $\alpha_3=(1,1,1)$ is a basis for $V=\mathbb{R}^3$ over $\mathbb{R}$. [5]

7. Examine if the set of vectors $\{(2,1,1),(1,2,2),(1,1,1)\}$ is linearly dependent in $\mathbb{R}^3$. [5]

8. Find a basis for the vector space $\mathbb{R}^3$ over $\mathbb{R}$, that contains the vectors $(1,2,0)$ and $(1,3,1)$. [5]

9. Let $\alpha_1=(1,2,0)$, $\alpha_2=(3,-1,1)$, $\alpha_3=(4,1,1)$. Show that the set $S=\{\alpha_1,\alpha_2,\alpha_3\}$ is linearly dependent. Apply deletion theorem to find a proper subset of S that can generate L(S). [5]

10. Let $\{\alpha,\beta,\gamma\}$ be a basis of real vector space V and *c* be a non-zero real number, prove that $\{\alpha+c\beta, \beta,\gamma\}$ is a basis of V. [5]

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